

# 3.4 Trigonometric Proof

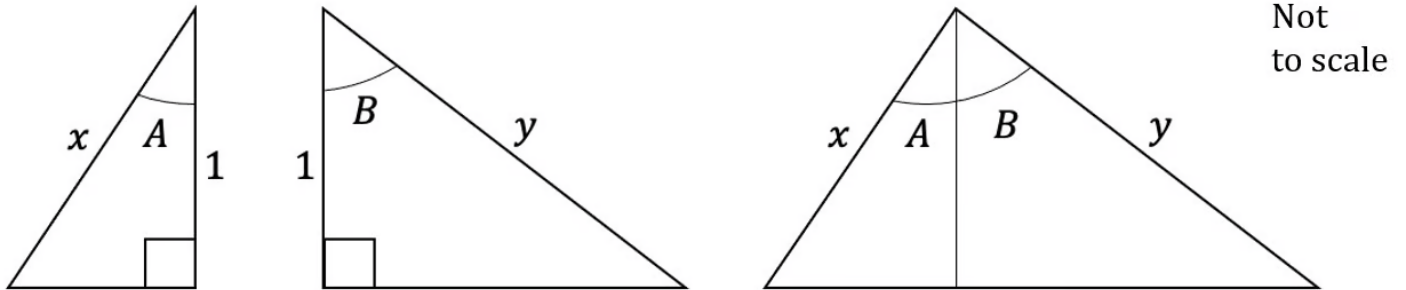
## Question Paper

Course	CIEA Level Maths
Section	3. Trigonometry
Topic	3.4 Trigonometric Proof
Difficulty	Very Hard

**Time allowed:** 60  
**Score:** /48  
**Percentage:** /100

## Question 1

Consider the three triangles, all of height 1, as shown below.



By applying the area of a triangle formula  $A = \frac{1}{2}ab \sin C$  to each one, prove that,

$$\sin(A + B) \equiv \sin A \cos B + \sin B \cos A$$

Briefly explain why this only proves the result for  $A$  and  $B$  being acute angles.

[6 marks]

**Question 2**

Prove the identity

$$\tan 4\theta \equiv \frac{4 \tan \theta (1 - \tan^2 \theta)}{1 - 6 \tan^2 \theta + \tan^4 \theta}$$

[4 marks]

**Question 3**

Prove the identity

$$-16 \cot 2\theta \operatorname{cosec}^3 2\theta \equiv \sec^4 \theta - \operatorname{cosec}^4 \theta$$

[5 marks]

**Question 4**

Show that

$$\frac{\sqrt{2} \cos\left(\theta + \frac{\pi}{4}\right)}{\sin\left(\theta - \frac{\pi}{2}\right)} \equiv \tan \theta - 1$$

[4 marks]

**Question 5a**

(a) Show that

$$\sin 3\theta \equiv 3 \sin \theta \cos^2 \theta - \sin^3 \theta$$

[4 marks]

**Question 5b**

(b) Hence, or otherwise, show that

$$\frac{\cos 3\theta - \cos \theta}{\sin 3\theta \sin \theta} \equiv \frac{4 \cos \theta}{1 - 4 \cos^2 \theta} \quad \theta \neq k\pi$$

[5 marks]

**Question 6**

Show that

$$4\cos^2\left(x - \frac{\pi}{6}\right) \equiv 3 - 2\sin^2 x + \sqrt{3}\sin 2x$$

[5 marks]

**Question 7**

Show that

$$\tan\left(\frac{2x + \pi}{4}\right) \equiv \sec x + \tan x$$

[6 marks]

**Question 8**

Show that

$$\frac{1}{\left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta\right)^2} \equiv 4 \operatorname{cosec}^2\left(2\theta + \frac{\pi}{3}\right)$$

**[9 marks]**